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Field-Enlarging Transformations and Chiral Theories

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A field-enlarging transformation in chiral electrodynamics is performed. This introduces an additional gauge symmetry that is unitary and anomaly free and allows for comparison of different models discussed in the literature. The problem of superfluous degrees of freedom and their influence on quantization is discussed. Several so-called mysteries are explained from this point of view.

Consistent quantization of an anomalous chiral gauge theory has long been problematic. In several simple cases, physically consistent and unitary models can be obtained (Jackiw and Rajaraman, 1985; Mitra, 1992; Sładkowski, 1992; Babelon *et al.*, 1986; Gomis and Paris, 1993; Braga and Montani, 1991; De Jonghe *et al.*, 1993). But it still remains one of the most important open questions in field theory (Gomis and Paris, 1993; Abdelhafiz *et al.*, 1986; Sładkowski and Zrałek, 1992). To solve the problem one usually adds in a more or less sophisticated way additional terms to the Lagrangian (Jackiw and Rajaraman, 1985; Mitra, 1992; Sładkowski, 1992; Babelon *et al.*, 1986; Gomis and Paris, 1993; Braga and Montani, 1991; De Jonghe *et al.*, 1993; Faddeev and Shatashvili, 1986; Rajeev, 1986; Harada and Tsutsui, 1987). Another way is to introduce a nonlocal gauge-fixing or interaction term (Thompson and Zhang, 1987; Della Serva *et al.*, 1993; Demarco *et al.*, 1992). The resulting theory is then invariant with respect to a restricted gauge symmetry that is not anomalous. Here we apply a field-enlarging transformation to analyze the problem (Sładkowski, 1992; Alfaro and Damgaard, 1990, 1992; Hosoya and Kikkawa, 1975). This transformation introduces additional scalar degrees of freedom to the system and restores gauge symmetry, although not always the one one started with. It is then possible to show explicitly

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the relations among various proposals and how the mechanism works. The conventional common part of the Lagrangian for the models discussed in the literature (chiral electrodynamics) is

$$L = -\frac{1}{4} F^{\mu\nu}F_{\mu\nu} + \bar{\psi}\left[i\partial^{\mu}\gamma_{\mu} - \frac{e}{2}(1 + \gamma^5)A_{\mu}\gamma^{\mu} \right]\psi \tag{1}$$

This Lagrangian is invariant with respect to

$$\delta A_{\mu} = \partial_{\mu}\alpha \tag{2a}$$

$$\delta\psi = -i\alpha \frac{e}{2}(1 - \gamma^5)\psi \tag{2b}$$

$$\delta\bar{\psi} = i\frac{\alpha e}{2}\bar{\psi}(1 + \gamma^5) \tag{2c}$$

where α is an arbitrary real function. Unfortunately, this gauge invariance is spoiled at the quantum level (Bell and Jackiw, 1969; Adler, 1969). Let us perform the following field-enlarging transformation (Sładkowski, 1992; Alfaro and Damgaard, 1990, 1992; Hosoya and Kikkawa, 1975):

$$A_{\mu} \to A_{\mu} - \partial_{\mu}\phi \equiv g_{\mu}(A, \phi) \tag{3}$$

in the Lagrangian (1). The transformed Lagrangian has the form

$$L = -\frac{1}{4} F^{\mu\nu}F_{\mu\nu} + \bar{\psi}\left[i\partial^{\mu}\gamma_{\mu} - \frac{e}{2}(1 + \gamma^5)(A_{\mu}\gamma^{\mu} - \gamma^{\mu}\partial_{\mu}\phi) \right]\psi \tag{4}$$

Although this seems to be trivial at first sight, especially when the gauge field mass term and/or gauge-fixing term for the symmetry (2) are absent, the consequences are not (Sładkowski, 1992; Alfaro and Damgaard, 1990, 1992; Hosoya and Kikkawa, 1975). The reason is that that quantization of a chiral fermion results in a nontrivial interaction that breaks the classical gauge symmetry (anomaly). It is also possible to redefine the fermion field via

$$\psi \to e^{f(\phi,\gamma^5)}\psi$$
$$\bar{\psi} \to e^{f^{\dagger}(\phi,\gamma^5)}\bar{\psi}$$

Then the fermion field is not invariant with respect to (5). In fact, it is also possible to choose the function f so that the scalar field ϕ is absent from the Lagrangian (4). But then one should worry about the Jacobian in the fermionic sector. We have chosen the simplest field redefinition so that everything is explicit! The transformation (6) introduces the following additional Abelian gauge symmetry into the theory (Sładkowski, 1992; Alfaro and Damgaard, 1990, 1992; Hosoya and Kikkawa, 1975):

$$\delta\phi(x) = \overline{\alpha}(x) \tag{5a}$$

$$\delta A_\mu(x) = -\int d^n x\, d^n y \left(\frac{\delta g^\mu(A, \phi)}{\delta A_\nu}\right)^{-1} (x, y) \frac{\delta g_{nu}(A, \phi)}{\delta\phi} \overline{\alpha}(z) = \partial_\mu\overline{\alpha}(x) \tag{5b}$$

$$\delta\psi = \overline{\psi} = 0 \tag{5c}$$

where $\overline{\alpha}$ is an arbitrary real function. To quantize this model we have to fix both gauge symmetries (Kugo and Ojima, 1979).

Now we are prepared to analyze the problem of quantization of an anomalous chiral gauge theory. Thompson and Zhang (1987) and Della Serva *et al.* (1993) proposed to perform the nonlocal transformation

$$A_\mu \to A_\mu^g = A_\mu - \partial_\mu \frac{1}{\Box} \partial_\nu A^\nu \tag{6}$$

in (1). The resulting theory

$$L = -\frac{1}{4} F^{\mu\nu}F_{\mu\nu} + \overline{\psi}\left[i\partial^\mu\gamma_\mu - \frac{e}{2}(1 + \gamma^5)A_\mu^g\gamma^\mu \right]\psi \tag{7}$$

is then invariant with respect to

$$\delta A_\mu = \partial_\mu\alpha \tag{8a}$$

$$\delta\psi = \delta\overline{\psi} = \delta A_\mu^g = 0 \tag{8b}$$

This symmetry is anomaly free because the fermion field transforms in a trivial way (Fujikawa, 1980). It can be shown that such nonlocal theories are unitary and consistent (Thompson and Zhang, 1987; Della Serva *et al.*, 1993; Demarco *et al.*, 1992). Unfortunately, these conclusions usually concern the additional gauge symmetry that has been introduced into the theory in question, but not the one we started with. The above model is still anomalous with respect to the original $U(1)$ gauge symmetry. Such a Lagrangian might yield a physically acceptable theory, but this is far from being the rule (Jackiw and Rajaraman, 1985). We should get rid of the anomalous symmetry. The simplest solution is the following. Let us try to quantize the model given by equation (4). First, let us remove the original (classical!) gauge symmetry (2) by the nonlocal gauge-fixing condition

$$\phi - \frac{1}{\Box} \partial_\mu A^\mu = 0 \tag{9}$$

The Lagrangian has the form (we omit the Faddeev–Popov ghost term)

$$L = -\frac{1}{4} F^{\mu\nu}F_{\mu\nu} + \bar{\psi}\left[i\partial^{\mu}\gamma_{\mu} - \frac{e}{2}(1 + \gamma^5)(A_{\mu}\gamma^{\mu} - \gamma^{\mu}\partial_{\mu}\phi) \right]\psi \qquad (10)$$

$$+ \rho\left(\phi - \frac{1}{\Box} \partial_{\mu}A^{\mu} \right)$$

where an auxiliary scalar field ρ has been introduced to exponentiate the functional Dirac δ-function that forces the gauge condition (9). Now we can perform the path integral over the scalar fields. This results in

$$L = -\frac{1}{4} F^{\mu\nu}F_{\mu\nu} + \bar{\psi}\left[i\partial^{\mu}\gamma_{\mu} - \frac{e}{2}(1 + \gamma^5)\left(A_{\mu}\gamma^{\mu} - \partial_{\nu}\gamma^{\nu}\left(\frac{1}{\Box} \partial_{\mu}A^{\mu} \right) \right) \right]\psi \qquad (11)$$

This is the Lagrangian given by (7) (Thompson and Zhang, 1987) with the A^g field written explicitly! The additional gauge symmetry (8) is the same as (5). Of course, other gauge conditions lead to different representations of the model. This shows that the proposal put forward in Thompson and Zhang (1987) and Della Serva *et al.* (1993) is to break the original symmetry (2) and to introduce a new one that is anomaly free (and in some sense trivial because it leaves fermions invariant). In fact, it can be shown that the transformation (6) chooses the covariant gauge

$$\partial_{\mu}A^{\mu} = 0 \qquad (12)$$

So we should not speak of a transformation but rather of a gauge-fixing condition. More sophisticated gauge conditions breaking (5) would result in more complicated Lagrangians.

Jackiw and Rajaraman (1985), in their seminal paper, discovered that the two-dimensional chiral Schwinger model yields a consistent and unitary, although anomalous and not gauge-invariant, theory. Following this, several other consistent anomalous models were put forward. They have the following general form:

$$L = -\frac{1}{4} F^{\mu\nu}F_{\mu\nu} + \bar{\psi}\left[i\partial^{\mu}\gamma_{\mu} - \frac{e}{2}(1 + \gamma^5)A_{\mu}\gamma^{\mu} \right]\psi$$

$$+ \frac{1}{2}B^2 - B\partial_{\mu}A^{\mu} + \partial_{\mu}\bar{c}\partial^{\mu}c + m^2K(\phi, A) + \phi P(A) \qquad (13)$$

where B, c, and P denote the auxiliary field that linearizes the gauge condition, the appropriate ghosts, and the Pontryagin term (Jackiw, 1984), respectively. Several forms of the K term have been discussed in the literature (Sładkowski, 1992; Jackiw and Rajaraman, 1985; Mitra, 1992; Babelon *et al.*, 1986; Gomis and Paris, 1993; Braga and Montani, 1991; De Jonghe *et al.*, 1993; Faddeev and Shatashvili, 1986; Rajeev, 1986; Harada and Tsutsui, 1987; Thompson and Zhang, 1987). In the (1 + 1)-dimensional case, it is possible to calculate the functional integral over the fermions (Jackiw, 1984) in (1). Then one can apply the transformation (3) (Sładkowski, 1992). This leads (after "reintroduction" of fermions and addition of the gauge-fixing and ghost terms) to

$$m^2 = \frac{e^2}{4\pi}(a - 1) \tag{14a}$$

where a is the quantization (regularization) ambiguity parameter (Jackiw and Rajaraman, 1985) and

$$K(\phi, A) = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \partial_\mu\phi A^\mu \tag{14b}$$

This form corresponds to a theory that possesses the additional gauge symmetry (5). This additional symmetry is the unexpected gauge invariance discovered by Harada and Tsutsui (1987) after adding the Wess–Zumino term to the chiral electrodynamic Lagrangian. This form of the K term has been recognized by Della Serva *et al.* (1993) as the one corresponding to the model discussed by Jackiw and Rajaraman (1985). This is not so, because the additional symmetry is absent in their model (Sładkowski, 1992). One has to break the additional symmetry in order to get the Jackiw and Rajaraman model (Sładkowski, 1992). Faddeev and Shatashvili (1986) chose $K = 0$. This corresponds to the $a = 1$ case of (14a). Path integration over the scalar field ϕ leads to the condition $P = 0$, which ensures the invariance with respect to (5). The form proposed by Rajeev (1986),

$$K(\phi, A) = \frac{1}{2}(\partial_\mu\phi - A_\mu)(\partial^\mu\phi - A^\mu) \tag{15}$$

has an additional term $\frac{1}{2}e^2A_\mu A^\mu$ that breaks the symmetry (8). It should be interpreted as a mass term for the gauge boson [Stückelbeg formalism (Sładkowski, 1993)]. Finally, Thompson and Zhang proposed to take

$$K(\phi, A) = \partial_\mu\rho(\partial^\mu\phi - A^\mu) \tag{16}$$

where ρ is an auxiliary scalar field. This model is equivalent to the ordinary chiral Schwinger model (Jackiw and Rajaraman, 1985; Sładkowski, 1992). This can be seen by integrating over the scalar fields. Note that this differs from (4) (Thompson and Zhang, 1987) in that the additional symmetry is broken. This shows once more that for any value of a the original gauge symmetry is lost in the quantization process (Jackiw and Rajaraman, 1985;

Jackiw, 1984). The above analysis shows that the consistency of the quantization of the discussed models has common roots that have been discovered by Jackiw and Rajaraman (1985) because the differences in the K terms can be regarded as different gauge-fixing terms for the symmetry (5). Note that the discussed Lagrangians can be obtained also in more sophisticated ways (Babelon *et al.*, 1986; Gomis and Paris, 1993; Braga and Montani, 1991; De Jonghe *et al.*, 1993; Alfaro and Damgaard, 1993). The important fact is that the additional symmetry (5) reveals itself in every case, although it might not be obvious, e.g., in the field–antifield formalism it is fixed in the due process (Gomis and Paris, 1993; Sładkowski, 1993; Alfaro and Damgaard, 1993; Fujiwara *et al.*, 1990).

An important question arises: *Can a field-enlarging transformation help to construct a nontrivial anomaly-free theory?* The answer may be affirmative. It has been observed that a theory can possess a BRST symmetry (Becci *et al.*, 1974; Tyutin, 1975) that is not a symmetry of the Lagrangian but only of the functional integral. This means that several symmetries, if "broken correctly," may result in an anomaly-free subsymmetry (cancellation of the anomalous terms in the fermionic determinant). To shed more light on the problem, let us consider the BRST symmetries that correspond to (2) and (5). The general formula for a BRST current associated to the fields that appear in (13) is (Della Serva *et al.*, 1993; Kugo and Ojima, 1979; Becci *et al.*, 1974; Tyutin, 1975)

$$J^{\mu}_{\text{BRST}} = F^{\mu\nu}\partial_\nu c - \phi \frac{\partial P}{\partial \partial_\mu A_\nu} \partial_\nu c + eJ^{\mu}_{L} c + B\partial^{\mu} c - \frac{\partial K}{\partial \partial_\mu \phi} \delta_{\text{BRST}}\phi \tag{17}$$

where J_L denotes the left fermion current. Its divergence is

$$\partial J_{\text{BRST}} = e\partial J_L c - P\delta_{\text{BRST}}\phi - \left(\frac{\partial K}{\partial \partial A_\mu} \partial_\mu c + \frac{\partial K}{\partial \partial_\mu \phi} \partial_\mu \delta_{\text{BRST}}\phi\right) \tag{18}$$

so that if K is gauge invariant and $\delta_{\text{BRST}}\phi = c$, J^{μ}_{BRST} is conserved ($\partial J_{\text{BRST}} = 0$) (Thompson and Zhang, 1987). It is obvious that this condition is fulfilled by the K-terms given by (15) and (16). This conserved BRST charge corresponds to the diagonal part of (2) and (5) ($\alpha = \overline{\alpha}$). The form of the K-term given by (14) (and its special case $K = 0$) defines a model that is gauge invariant with respect to (5) and the appropriate BRST current is also conserved. Unfortunately, the above phenomenon seems to require additional fields or/and nonlocal terms. It is also obscure whether, and to what extent, it can work in higher than (1 + 1)-dimensional spacetime. The Batalin–

Vilkovisky or the field-enlarging [Stückelberg (Sładkowski, 1993)] formalism discussed here should be helpful in analyzing this problem. Especially, the role of the additional symmetry should be explored. This problem is under investigation. Recently, similar ideas have been discussed in the context of W_2-*gravity* (De Jonghe *et al.*, 1993).

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